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THERMAL AND ROTATIONAL EVOLUTION OF NEUTRON STARS WITH INTERNAL VORTEX CREEP

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Abstract

We have studied the thermal and rotational evolution of the neutron star. In order to describe the internal dynamical behavior we have considered the creep motion of the pinned superfluid vortices in the inner crust. The internal heat generation accompanied by this vortex creep motion has also been included in the thermal behavior. It is shown that the evolutionary course of the star is characterized by the thermal and dynamical equilibria or disequilibria. Via linear analysis we have found that the neutron star in the thermal and dynamical equilibrium state is thermally unstable below a certain critical temperature. This critical temperature is determined by the condition that in the unstable regime the thermal time is shorter than the dynamical time. It is explained that this thermal instability arises from the fact that the vortex creep motion is accelerated or decelerated considerably by the slight change in temperature. We discuss the onset time of this thermal instability taking into account the decay of the magnetic field.

I. INTRODUCTION

The neutron star is expected to contain the superfluid in its interior. Since the neutron star rotates, the vortex line parallel to the rotation axis is formed in the superfluid. The slow relaxation of the pulse frequency after the glitch (sudden jump of the pulse frequency in pulsars) has been thought to reflect the weak coupling of the neutron superfluid to the charged components. The simple two-component model star with the superfluid core and the solid crust (Baym *et al.* 1969) provided a qualitative explanation for the post-glitch behavior in terms of the coupling between the two components.

The core-crust coupling gives rise to the frictional heat generation, which in turn affects the coupling strength itself. Hence, the neutron star is a thermally and dynamically coupled system. Greenstein (1975) and Harding, Guyer and Greenstein (1978) examined the effect of the frictional heating on the thermal history of the star and predicted that all the long-period pulsars may have essentially the same surface temperature, apart from its variation with mass. The stability analysis done by Greenstein (1979) indicated that the frictional heat generation may give rise to a thermal instability. Recently, Shibazaki and Lamb (1986a) also considered the thermal and rotational evolution of the frictionally coupled two-component star examining the stability of the equilibrium states that the star may undergo in its evolutionary course. They found that the neutron star is thermally unstable below a certain critical temperature if the core-crust coupling time has an exponential dependence on temperature as in the case of the electron scattering from the vortex excitation (Feibelman 1971). Whereas, if the coupling time is not dependent on or only weakly depends on the temperature, as in the case of electron scattering from the magnetized vortex (Sauls, Stein and Serene 1982; Alpar, Langer and Sauls 1984), the neutron star was shown to evolve without the thermal and dynamical instability.

The timing noise analysis of the Crab pulsar (Boynton 1981) and Vela X-1 (Boynton *et al.* 1984) revealed that the neutron star is responding to rotational disturbances like a rigid body. Furthermore, Alpar, Langer and Sauls (1984) showed that the $^3P_2$ neutron superfluid vortex in the core is strongly magnetized due to the induced proton current and couples to the charged components and hence the crust in a very short time. These results indicate that the simple two-component model is not an adequate description of the full dynamical behavior of the neutron star.

It is expected that the neutron superfluid in the inner crust is in the $^1S_0$ pairing state and the superfluid vortices are pinned to the lattice nuclei present there. Recently, Alpar et. al (1984a) considered the dynamics of a neutron star taking account of the pinned superfluid neutrons and developed a general theory for the vortex motion thermally activated against the pinning barrier. They interpreted the glitch as an unpinning event of the superfluid vortices in the inner crust (Anderson and Itoh 1975) and explained the observed post-glitch behaviors excellently as resulting from the recoupling of the pinned superfluid to the rest of the star (Alpar *et al.* 1984b; Alpar, Nandkumar and Pines 1985).

The outward vortex creep motion accompanies the internal heat generation. Recently, Shibazaki and Lamb (1986*b*; hereafter referred to as Paper I) studied the cooling of the neutron star, taking into account internal heating by this vortex creep motion. They found that the thermal evolution in the photon cooling era is profoundly affected by the internal heating. Hence, this study indicates that the thermal and dynamical behaviors need to be combined in order to understand the evolution of the neutron star.

In this paper we consider the thermal and dynamical evolution of the neutron star, noting the stability of the equilibrium state. We assume that the internal dynamical behavior of the neutron star is determined by the vortex creep motion. In §II we outline the vortex creep model and present the basic equations to describe the thermal and dynamical behaviors of the star. In §III the possible evolutionary course is sketched in terms of the characteristic equilibrium states. In §IV the stability is examined on the thermal and dynamical equilibrium. In §V the physical meaning of the instability found in §IV is explained. In the last section we discuss the evolution of the neutron star, comparing the results here with the cooling curves calculated in Paper I.

II. BASIC EQUATIONS IN THE VORTEX CREEP CASE

The neutron in the inner crust is $^1S_0$ superfluid, and the superfluid vortices are pinned to the lattice of nuclei present there since the energy cost per particle to create the normal core of the vortex line is reduced by the pinning. The pinned vortex line rotates together with the nucleus lattice at the angular velocity of the crust. The superfluid in the core also rotates together with the crust since the strong magnetization of the core vortex line yields an instantaneous coupling of the core to the crust (Alpar, Langer and Sauls 1984). Hereafter, the crust refers to the rest of the neutron star except for the inner crust. If there is a relative motion between the crust and the superfluid in the pinning site,

$$\omega = \Omega_p - \Omega_c \quad , \tag{1}$$

the Bernoulli force acts on the pinned vortex line. Here Ω_p and Ω_c are the angular velocities of the superfluid in the pinning region and the crust,respectively, and ω is the angular velocity lag. When $\omega > 0$, the Bernoulli force produces the bias that statistically makes the motion of the thermally activated vortices outward. The vortex creep motion transfers the angular momentum from the inner crust to the rest of the star. Consequently, the rotation of the inner crust is decelerated and the rotational energy is dissipated into heat.

2) Basic equations

We assume uniform rotation for both the pinning region and the crust and isothermality throughout the star. The rotational behavior of the crust is determined by

$$I_c\dot{\Omega}_c = N_{ext} + N_{int} \ , \tag{2}$$

where I_c is the moment of inertia of the crust and N_{ext} and N_{int} are the external and internal torques exerting on the crust, respectively. According to a vortex creep theory by Alpar *et al.* (1984a), the internal torque due to the vortex creep motion is given by

$$N_{int} \equiv -I_p\dot{\Omega}_p = I_p \frac{2V_0}{r_p} \Omega_p \exp\left[-\frac{E_p}{kT}(1-\omega/\bar{\omega}_{cr})\right] \quad , \tag{3}$$

where I_p is the moment of inertia of the pinning region, r_p the radial distance of the pinning site ($r_p \sim 10^6$ cm), V_0 the velocity of the microscopic motion of the vortex line ($V_0 \sim 10^7$ cm/s), E_p the average pinning energy ($E_p \sim 1$ MeV), T the temperature and k the Boltzmann constant. Here ω_{cr} is the appropriate average of the critical velocity lag at which the Bernoulli force on the vortex line exceeds the pinning force. The value of ω_{cr} strongly depends on the pinning condition in the inner crust. ω_{cr} = 10–20 if the pinning is strong, ω_{cr} = 0.1 –1 if weak, and $\omega_{cr} < 0.1$ if superweak (see Alpar et al. 1984 b). The value of I_p depends on the neutron star model, stiff or soft, and is estimated as $I_p/I \sim 2.5\times10^{-3}$ for the soft star, $I_p/I = 5\times10^{-3} - 2\times10^{-2}$ for the moderately stiff satr and $I_p/I = 0.14$ for the stiff star (Nandkumar 1985). Here I is the total moment of inertia given by $I = I_p + I_c$. The comparison of the vortex creep model with the glitch observations suggests $I_p/I \sim 0.01$ (Alpar *et al.* 1984*b*; Alpar, Nandkumar and Pines 1985).

The energy equation is written as

$$C_V\dot{T} = H - \Lambda \quad , \tag{4}$$

where C_v is the heat capacity, H the internal heating rate and Λ the cooling rate. The heat generation rate due to the vortex creep motion is given by

$$H = -I_p \omega \dot{\Omega}_p \quad . \tag{5}$$

When the neutron star is hot, it cools by the emission of the neutrino. At lower temperature the photon emission from the surface becomes more important. The cooling rate by these processes is well approximated by

$$\Lambda = BT^n \quad , \tag{6}$$

where B and n are constants that depend on the cooling mechanism. In the temperature range of interest, the value of n is in the range of 2 to 6 (Baym and Pethick 1979; Gudmundsson, Pethick and Epstein 1983). The heat capacity for the degenerate matter is expressed as

$$C_V = aT \quad , \tag{7}$$

where a is the constant. The main contribution for the heat capacity is from normal neutrons and electrons.

III. CHARACTERISTIC EVOLUTIONARY STAGES

There are two equilibria conceivable in the course of the evolution, dynamical and thermal equilibria. The evolutionary stage can be characterized by the thermal and dynamical equilibria or disequilibria.

1) Dynamical and thermal equilibria

If the system of pinned superfluid and crust that are coupled through the vortex creep motion is in dynamical equilibrium, both components decelerate at the same rate:

$$\frac{N_{exto}}{I} \equiv \dot{\Omega}_{co} = \dot{\Omega}_{po} \equiv -\frac{2V_0}{r_p}\Omega_{po} \exp\left[-\frac{E_p}{kT_o}(1-\omega_o/\bar{\omega}_{cr})\right] . \tag{8}$$

Hereafter, the subscript "o" denotes the equilibrium state value. In order to estimate the equlibrium lag in angular velocity, we rewrite Eq. (8) as

$$1-\frac{\omega_o}{\bar{\omega}_{cr}} = \ln\left(2\frac{\Omega_{po}}{|\dot{\Omega}_{po}|}\frac{V_0}{r_p}\right)\frac{kT_o}{E_p} \equiv \beta\frac{kT_o}{E_p} . \tag{9}$$

In the situation with which we are concerned, $kT_o < 10$ keV and $E_p \sim 1$ MeV while the logarithmic factor β is in the range of 10 to 40. Hence, we can see that in dynamical equilibrium the angular velocity lag is very close to the critcal value:

$$\omega_o \sim \bar{\omega}_{cr} . \tag{10}$$

In connection with the post-glitch behavior of a pulsar, Alpar *et al.* (1984*a*) studied the response of the system of pinned superfluid and crust coupled through the vortex creep motion when the sudden jump in angular velocity is introduced into the dynamical equilibrium state. They found that the dynamical disturbance decays with time, which shows that the dynamical equilibrium state is stable. The dynamical relaxation time due to the vortex creep is obtained as

$$\tau_{dr}=\frac{\bar{\omega}_{cr}}{\Omega_{co}}\frac{kT_o}{E_p}\tau_{so} \quad , \tag{11}$$

where τ_s is the spin-down time defined by

$$\tau_s=\frac{\Omega_c}{|\dot{\Omega}_c|} \quad . \tag{12}$$

It is to be noted that the dynamical relaxation time here corresponds to the coupling time of the core to the crust in a simple two-component model (Shibazaki and Lamb 1986a). Hence, the dynamical equilibrium is possible when

$$\tau_{dr} << \tau_{so} \quad . \tag{13}$$

Thermal equilibrium refers to the state where the heating and cooling rates are in balance:

$$-I_p\omega_o\dot{\Omega}_{po}=BT_o^n \quad . \tag{14}$$

The time scale that characterizes the thermal behavior is the cooling (or heating) time defined by

$$\tau_{cool}=\frac{C_VT_o}{BT_o^n}\equiv\frac{C_VT_o}{I_p\omega_o|\dot{\Omega}_{po}|} \quad . \tag{15}$$

In order for thermal equilibrium to be achieved, the cooling time needs to be much shorter than the spin-down time:

$$\tau_{cool} << \tau_{so} \quad . \tag{16}$$

2) Thermal and dynamical evolution

The neutron star at birth is considered to be very hot, with a temperature of ~ 10^{11} K, and it rotates rapidly as a rigid body. Initially, it cools extremely rapidly because of the high neutrino emissivity. When the thermal energy becomes less than the pinning energy,

$$T \lesssim \frac{E_p}{k} = 1.2\times 10^{10} \left(\frac{E_p}{1\ \mathrm{MeV}}\right) \quad K\ , \tag{17}$$

the neutron superfluid vortices in the inner crust start to be pinned to the lattice nuclei. The relative motion between the superfluid in the inner crust and the rest of the star is produced by the external torque, and then the system evolves to dynamical equlibrium through the vortex creep motion. The cooling calculations in Paper I show that if the pinning is weak or superweak depending on the neutrino emission mechanism, in the neutrino cooling era the heat generated by the vortex creep is negligible compared to the heat content of the star. The neutrino emission is far dominant over the internal heat generation. Hence, the star is not in thermal equilibrium. Since the dynamical equilibrium itself is stable, this stage persists until before the photon cooling era.

In the weak or superweak pinning regime (depending on the neutrino emissivity) the heat generation due to the vortex creep becomes the dominant energy source in the photon cooling era, while in the strong or weak pinning regime it controls the thermal evolution even from the neutrino cooling era. In this situation the thermal equilibrium is also possible in addition to the dynamical equilibrium. The star will cool and slow down, maintaining both equilibria if stable. If unstable, however, this stage will not persist even if it is once reached, and the subsequent evolution may be complicated.

It has been discussed in previous works (Greenstein 1979; Shibazaki and Lamb 1986*a*) that in the simple two-component case the crust decouples from the superfluid core at the later evolutionary stage with lower temperature when the coupling time becomes longer than the spin-down time. In the vortex creep case the decoupling condition is given by $\tau_{dr} > \tau_{so}$, which together with Eq. (11) yields

$$\Omega_{co} \lesssim 9\times10^{-12} \left(\frac{\overline{\omega}_{cr}}{0.1\ \mathrm{rad/s}}\right) \left(\frac{E_p}{1\mathrm{MeV}}\right)^{-1} T_o \quad \mathrm{rad\ s^{-1}}. \tag{18}$$

Equation (18) indicates that in reality there will be no decoupling between the crust and the pinned superfluid in the inner crust and the star, if stable, will slow down as a whole, maintaining the thermal and dynamical equilibrium.

The above discussion shows that the stability of the thermal and dynamical equilibrum state is quite important to obtain an understanding of the entire evolutionary course of the neutron star. In the next section we examine the stability of this state.

IV. STABILITY OF THE THERMAL AND DYNAMICAL EQUILIBRIUM STATE

We examine the stability of the thermal and dynamical equlibrium state determined by Eqs. (8) and (14).

1) Infinitesimal perturbation

In order to know the stability or instability, we perturb the equilibrium state and observe the subsequent variation of the perturbation:

$$\Omega_c = \Omega_{co} + \delta\Omega_c,\ \Omega_p = \Omega_{po} + \delta\Omega_p \text{ and } T = T_o + \delta T \quad , \tag{19}$$

where $\delta\Omega_c$, $\delta\Omega_p$ and δT are perturbations of angular velocities of crust and pinned superfluid and temperature, respectively. If the perturbation diminishes with time, we judge that the equilibrium state is stable and *vice versa.*

Inserting Eq. (19), the basic equations (2), (3) and (4) are linearized as

$$\frac{I_c}{I}\frac{\delta\dot{\Omega}_c}{\dot{\Omega}_{co}} = \left(\frac{\partial \ln|N_{ext}|}{\partial \ln\Omega_c}\right)_o \frac{\partial\Omega_c}{\Omega_{co}} - \frac{I_p}{I}\frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{po}} \tag{20}$$

$$\frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{po}} = -\frac{E_p}{kT_o}\frac{\Omega_{co}}{\bar{\omega}_{cr}}\frac{\delta\Omega_c}{\Omega_{co}} + \left(1+\frac{E_p}{kT_o}\frac{\Omega_{po}}{\bar{\omega}_{cr}}\right)\frac{\delta\Omega_p}{\Omega_{po}} + \frac{E_p}{kT_o}\left(1-\frac{\omega_o}{\bar{\omega}_{cr}}\right)\frac{\delta T}{T_o} \quad . \tag{21}$$

$$\frac{\delta\dot{T}}{\dot{T}_{cool}} = -\frac{\Omega_{co}}{\omega_o}\frac{\delta\Omega_c}{\Omega_{co}} + \frac{\Omega_{po}}{\omega_o}\frac{\delta\Omega_p}{\Omega_{po}} + \frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{po}} - n\frac{\delta T}{T_o} \quad , \tag{22}$$

where the external torque is assumed to be a function of Ω_c and T_{cool} is defined by

$$\dot{T}_{cool} \equiv \frac{T_o}{\tau_{cool}} \quad . \tag{23}$$

We consider the sinusoidal solution such as that given by

$$\frac{\delta\Omega_c}{\Omega_{co}} = \varepsilon_1 e^{-i\nu t}$$
$$\frac{\delta\Omega_p}{\Omega_{po}} = \varepsilon_2 e^{-i\nu t} \tag{24}$$
$$\frac{\delta T}{T_o} = \varepsilon_3 e^{-i\nu t}$$

where ε_1, ε_2, and ε_3 are the infinitesimal constants. We are interested in the solution that has the characteristic time much shorter than the spin-down time:

$$|\nu|^{-1} << \tau_{so} \quad . \tag{25}$$

Hence, we keep Ω_{co}, Ω_{po}, and T_o constant when we take a time derivative of the perturbation in Eq. (24).

Hereafter we neglect the external torque in Eq. (20) for simplicity since we found that its inclusion does not change the essential results on the instability. The neglect of the external torque term in Eq. (20) leads to the conservation of angular momentum:

$$I_c\delta\Omega_c + I_p\delta\Omega_p = 0 \quad . \tag{26}$$

The moment of inertia of the pinning region is suggested to be small as compared to the total from both the theoretical calculation and the observed post-glitch behavior of the pulsar. Here, we consider the case of

$$I_p << I \sim I_c \tag{27}$$

(We have also analyzed the case $I_p >> I_c$ and obtained essentially the same results). Equations (26) and (27) yield

$$|\delta\Omega_c| << |\delta\Omega_p| \quad , \tag{28}$$

which indicates that the $\delta\Omega_c$ terms in Eqs. (21) and (22) can be ignored compared to the $\delta\Omega_p$ terms.

Inserting Eq. (24) into Eqs. (21) and (22) and making use of the approximations mentioned above, we derive

$$\nu^2 + i\nu(\delta\tau_{dr}^{-1} - \xi\beta\tau_{cool}^{-1}) - n\varepsilon\tau_{dr}^{-1}\tau_{cool}^{-1} = 0 \quad , \tag{29}$$

where

$$\xi = 1 - \frac{n}{\beta} \tag{30}$$

$$\delta = 1 + \frac{kT_o}{E_p}\frac{\overline{\omega}_{cr}}{\Omega_{po}} \tag{31}$$

and

$$\varepsilon = \delta + \left(1 - \frac{\omega_o}{\overline{\omega}_{cr}}\right)\frac{\overline{\omega}_{cr}}{\omega_o}\frac{1}{n} \quad . \tag{32}$$

The factors ξ, δ, and ε are all of order one in the situation of interest in the current investigation.

2) Solutions

The property of the solutions to Eq. (29) is understood by examining the extreme cases.

(i) $\tau_{dr}/\delta >> \tau_{cool}/\xi\beta$

In this limit the first term in the bracket of Eq. (29) can be neglected. The two solutions obtained are:

$$\nu_1 \sim (\tau_{cool}/\xi\beta)^{-1}\, i \tag{33}$$

and

$$\nu_2 \sim \left(\frac{\xi\beta}{n\varepsilon}\tau_{dr}\right)^{-1} i \quad . \tag{34}$$

Since the imaginary part of ν is positive in both solutions, the perturbation grows with time, and hence the equilibrium state is unstable. It is to be noted that the growth times (~ $1/|\nu|$) of the perturbation in solutions ν_1 and ν_2 are related to the cooling and dynamical relaxation times, respectively. This evidence implies that the unstable modes ν_1 and ν_2 may be thermal and dynamical insabilities, respectively.

(ii) $\tau_{dr}/\delta << \tau_{cool}/\xi\beta$

Neglecting the second term in the bracket, Eq. (29) yields two solutions given by

$$\nu_1 \sim -\left(\frac{\delta}{n\varepsilon}\tau_{cool}\right)^{-1} i \tag{35}$$

and

$$\nu_2 \sim -\left(\frac{\tau_{dr}}{\delta}\right)^{-1} i \quad . \tag{36}$$

The negative imaginary part of these solutions indicates that the perturbation decays with time and hence the star is stable in this limit.

3) Instability criterion

It is known from the above results that the thermal and dynamical equilibrium state are unstable when the thermal time defined by $\tau_{th} = \tau_{cool}/\beta$ is shorter than the dynamical relaxation time:

$$\tau_{th}/\xi \equiv \tau_{cool}/\xi\beta < \tau_{dr}/\delta \quad . \tag{37}$$

Using Eqs. (7), (8), (10)-(12) and (15), Eq. (37) is rewritten, and the instability criterion is expressed in a more convenient way:

$$T_o < T_c \equiv 1.7\times10^5\, \xi\delta^{-1} \left(\frac{\beta}{40}\right) \left(\frac{E_p}{1\mathrm{MeV}}\right) \left(\frac{I_p}{10^{43}\mathrm{g\ cm}^2}\right) \bar{\omega}_{cr}^2 \quad \mathrm{K} \, . \tag{38}$$

It is now concluded that the thermal and dynamical equilibrium state of the neutron with the vortex creep motion in its interior is unstable when it cools down below a critical temperature T_c.

V. PHYSICAL INTERPRETATION OF THE INSTABILITY

We study the physical meaning of the two unstable modes found in the previous section.

1) Thermal response

First, we focus attention on the thermal behavior of the star when the temperature is perturbed. Under the condition of Eq. (37), the variation of the dynamical quantity can be neglected through the evolution of the temperatrue perturbation since the dynamical relaxation time is much longer than the thermal time. Neglecting the $\delta\Omega_c$ and $\delta\Omega_p$ terms in Eqs. (21) and (22) yields

$$\frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{p_o}} = \frac{E_p}{kT_o} \left(1 - \frac{\omega_o}{\overline{\omega}_{cr}}\right) \frac{\delta T}{T_o} \tag{39}$$

$$\frac{\delta\dot{T}}{\dot{T}_{cool}} = \frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{po}} - n\frac{\delta T}{T_o} \quad . \tag{40}$$

The solution to Eqs. (39) and (40) is calculated as

$$\delta T(t) = \delta T(0) \exp\left\{ t \,/\, \left(\frac{\tau_{th}}{\xi}\right) \right\} \quad . \tag{41}$$

The temperature perturbation grows with the time scale of τ_{th}/ξ.

The cause of this instability is easily understood as follows. Consider the initial increase in temperature as an input perturbation. According to Eq. (3), higher temperature produces larger internal torque. The internal heat generation is enhanced, which leads to further increase in temperature. Thus, the unstable mode ν_1 in the previous section is now identified as thermal instability.

2) Dynamical response

Next, we consider the dynamical perturbation maintaining the thermal equilibrium. Assuming thermal equilibrium and no external torque together with Eq. (27), Eqs. (20) ~ (22) reduce to

$$\frac{I_c}{I}\frac{\delta\dot{\Omega}_c}{\dot{\Omega}_{co}} = -\frac{I_p}{I}\frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{po}} \tag{42}$$

$$\frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{po}} = \left(1+\frac{E_p}{kT_o}\frac{\Omega_{po}}{\bar{\omega}_{cr}}\right)\frac{\delta\Omega_p}{\Omega_{po}} + \frac{E_p}{kT_o}\left(1-\frac{\omega_o}{\bar{\omega}_{cr}}\right)\frac{\delta T}{T_o} \tag{43}$$

$$\frac{\Omega_{po}}{\omega_o}\frac{\delta\Omega_p}{\Omega_{po}} + \frac{\delta\dot{\Omega}_p}{\dot{\Omega}_{po}} - n\frac{\delta T}{T_o} = 0 \quad . \tag{44}$$

Equations (42) ~ (44) can be solved to yield

$$\begin{aligned} \delta\Omega_c(t) &= \delta\Omega_c(0)\exp\left\{t\left(\frac{\xi\beta\tau_{dr}}{n\varepsilon}\right)^{-1}\right\} \\ \delta\Omega_p(t) &= -\frac{I_c}{I_p}\,\delta\Omega_c(0)\exp\left\{t\left(\frac{\xi\beta\tau_{dr}}{n\varepsilon}\right)^{-1}\right\} \end{aligned} \tag{45}$$

The perturbation grows approximately with a time scale of the dynamical relaxation time.

In order to understand the physical cause of this instability, consider the dynamical perturbation in which Ω_c increases and Ω_p decreases initially. The decrease in angular velocity lag accompanies the heat generation, which increases the temperature. The higher temperature produces the larger internal torque, which gives rise to further increase and decrease in $\Omega_c$and Ω_p, respectively. Thus, the unstable mode ν_2 found in the previous section is identified as the dynamical instability.

It should be noted that this dynamical instability, even if triggered, will be suppressed to proceed in reality because the thermal equilibrium assumed in the course of the perturbation is already unstable and thermal instability develops much faster than dynamical instability.

VI. DISCUSSION AND CONCLUDING REMARKS

We have found that the thermal and dynamical equilibrium state of the neutron star is stable above a certain critical temperature, but unstable below it. If the pinning of vortices to the lattice nuclei in the inner crust is weak or superweak, this critical temperature is less than 10^6K. Hence, this critical moment is expected to be in the photon cooling era. Whereas, in the case of the stiff star with the strongly pinned crustal superfluid (though less likely as explained in Paper I), this critical temperature can be as high as $\sim 10^8$K at which the neutrino emission is the dominant cooling mechanism. Our cooling calculations in Paper I indicate that in every case the neutron star has already established the thermal balance between the cooling and heating rates well before this critical temperature. Therefore, it is quite likely that the neutron star approaches and reaches the critical temperature maintaining the thermal and dynamical equilibrium. The direct plunge into the unstable equilibrium state with temperature much lower than the critical value will not happen unless the initial temperature at birth is unreasonably low.

The time for the neutron star to reach the critical temperature depends on the pinning condition, the star model, and also the magnetic field. The cooling curve in Paper I indicates that in the strong pinning regime the star can reach the critical temperature at the age of as young as $\sim 10^3$y. The comparisons of the vortex creep theory with the observational results on the Vela Pulsar and nearby old pulsars (Alpar et al. 1984b; Paper I), however, suggest that the weak and/or superweak pinning regime is more likely, which is consistent with the recent theoretical study on the property of the crustal superfluid (Chen et al. 1985). Furthermore, the vortex creep theory, when combined with the glitch observations, indicates that the stiff star is less likely. If we take into account these facts, then, we expect that the star at the critical temperature may be older than 10^8y. in the case of no magnetic field decay (Paper I). If the magnetic field decays, on the other hand, the time at the critical temperature becomes much earlier because the star cools down steeply with the time constant of the field decay as discussd in Paper I. Taking into account the current estimates for the magnetic field decay by Lyne, Manchester and Taylor (1985) and Taam and van der Heuvel (1986b), we expect the thermal instability around 10^6–10^7 y.

How does the thermal instability grow and effect the subsequent evolution of the neutron start? In order to know this it is necessary to perform a detailed numerical calculation including the nonlinear effects. If this thermal instability has some distinct feature, then its observation will provide the important information regarding the interior properties and magnetic field of the neutron star.

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REFERENCES

Alpar, M. A., Anderson, P. W., Pines, D., and Shaham, J. 1984*a*, *Ap. J.*, **276**, 325.

______________________________. 1984*b*, *Ap. J.*, **278**, 791.

Alpar, M. A., Langer, S. A., and Sauls, J. A. 1984, *Ap. J.*, **282**, 533.

Alpar, M. A., Nandkumar, R., and Pines, D. 1985, *Ap. J.*, **288**, 191.

Anderson, P. W., and Itoh, N. 1975, *Nature*, **256**, 25.

Baym, G., Pethick, C. J., Pines, D., and Ruderman, M. W. 1969, *Nature*, **224**, 872.

Baym, G., and Pethick, C. J. 1979, *Ann. Rev. Astron. Astrophys.*, **17**, 415.

Boynton, P. E. 1981, in *IAU Symposium 95, Pulsar,* ed. R. Wielebinski and W. Sieber (Dordrecht: Reidel), p. 279.

Boynton, P. E., Deeter, J. E., Lamb, F. K., Zylstra, G., Pravdo, S. H., White, N. E., and Wood, K. 1984, *Ap. J. (Letters)*, **283**, L53.

Chen, J.M.C., Clark, J.W., Krotscheck, E., and Smith, R.A. 1985, preprint.

Feibelman, P. J. 1971, *Phys. Rev. D*, **4**, 1589.

Greenstein, G. 1975, *Ap. J.*, **200**, 281.

Greenstein, G. 1979, *Nature*, **277**, 521.

Gudmundsson, E. H., Pethick, C. J., and Epstein, R. I. 1983, *Ap. J.*, **272**, 286.

Harding, D., Guyer, R.A., and Greenstein, G. 1978, *Ap. J.*, **222**, 991.

Lyne, A.G., Manchester, R.N., and Taylor, J.H. 1985, *Mont. Not. R. Astr. Soc.*, **213**, 613.

Nandkumar, R. 1985, private communication.

Sauls, J. A., Stein, D., and Serene, J. W. 1982, *Phys. Rev. D*, **25**, 967.

Shibazaki, N., and Lamb, F. K. 1986*a*, preprint.

__________________. 1986*b*, preprint (Paper I).

Taam, R.E., and van der Heuvel, E.P.J. 1986, *Ap. J.*, **305**, 235.

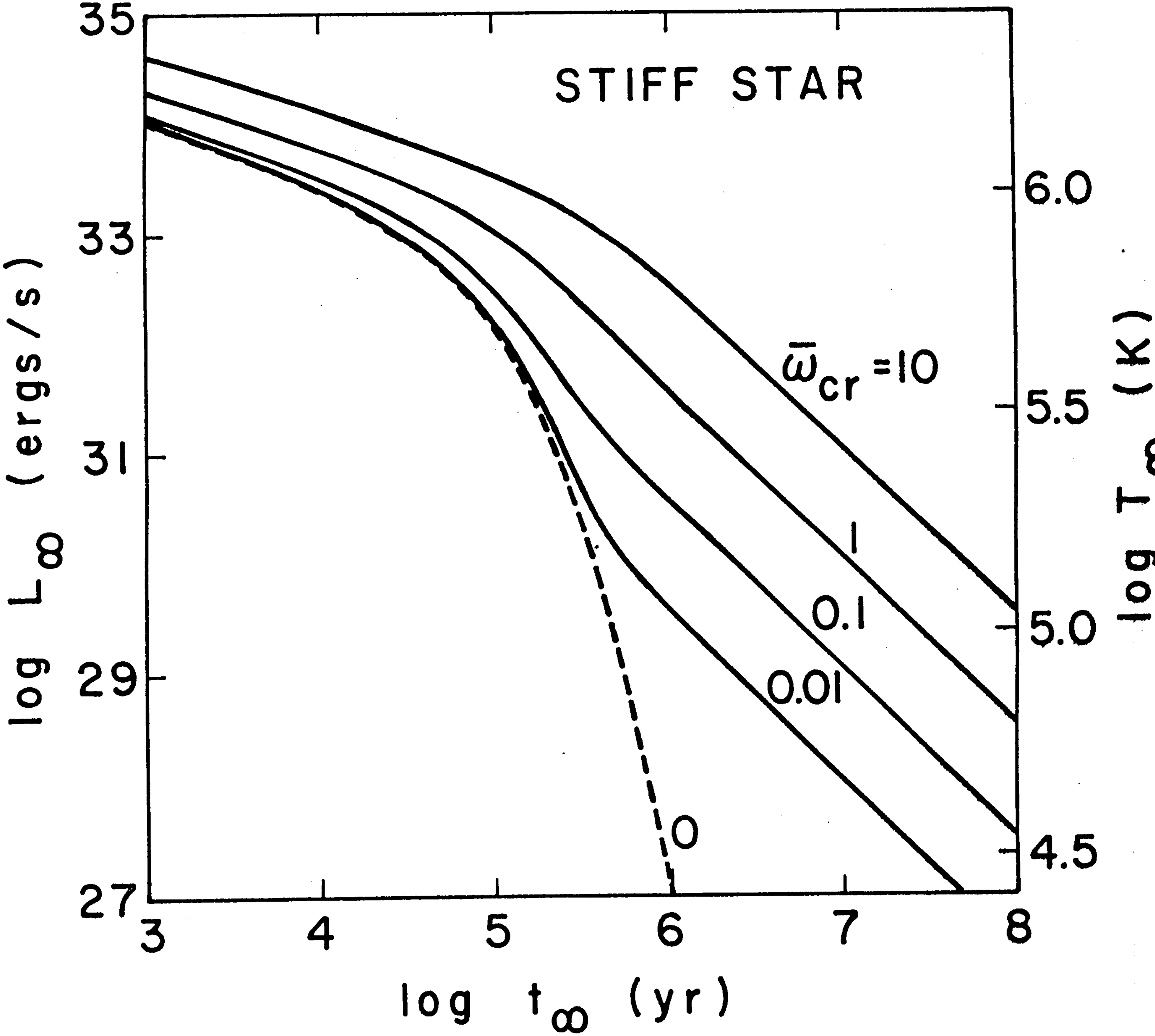
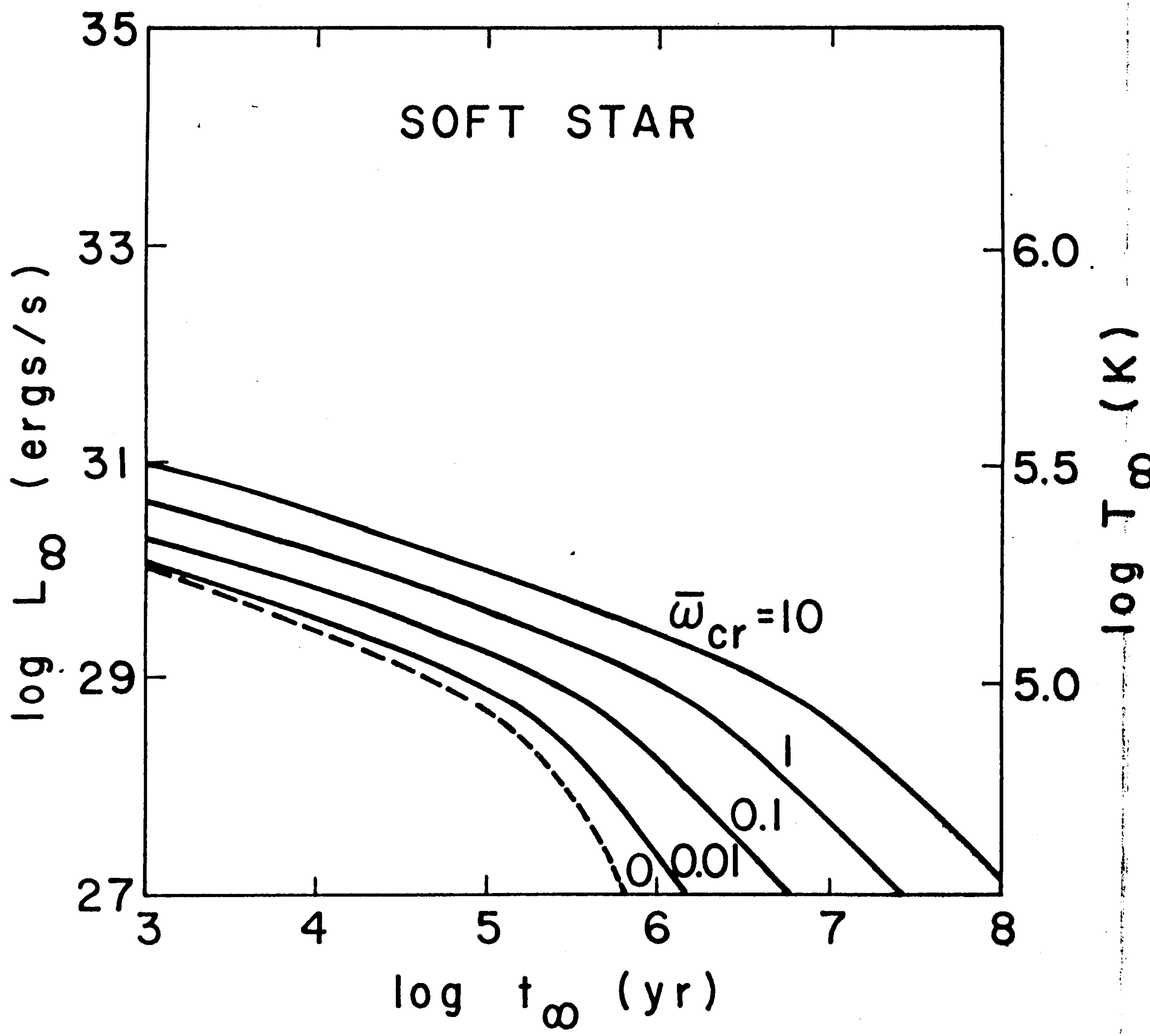

STIFF STAR
$\bar{\omega}_{cr}=10$
1
0.1
0.01
0
log L_∞ (ergs/s)
log T_∞ (K)
log t_∞ (yr)

Fig. 1



SOFT STAR
35
33
31
29
27
3
4
5
6
7
8
6.0
5.5
5.0
$\bar{\omega}_{cr}=10$
1
0.1
0.01
0
log L_∞ (ergs/s)
log t_∞ (yr)
log T_∞ (K)

Fig. 2

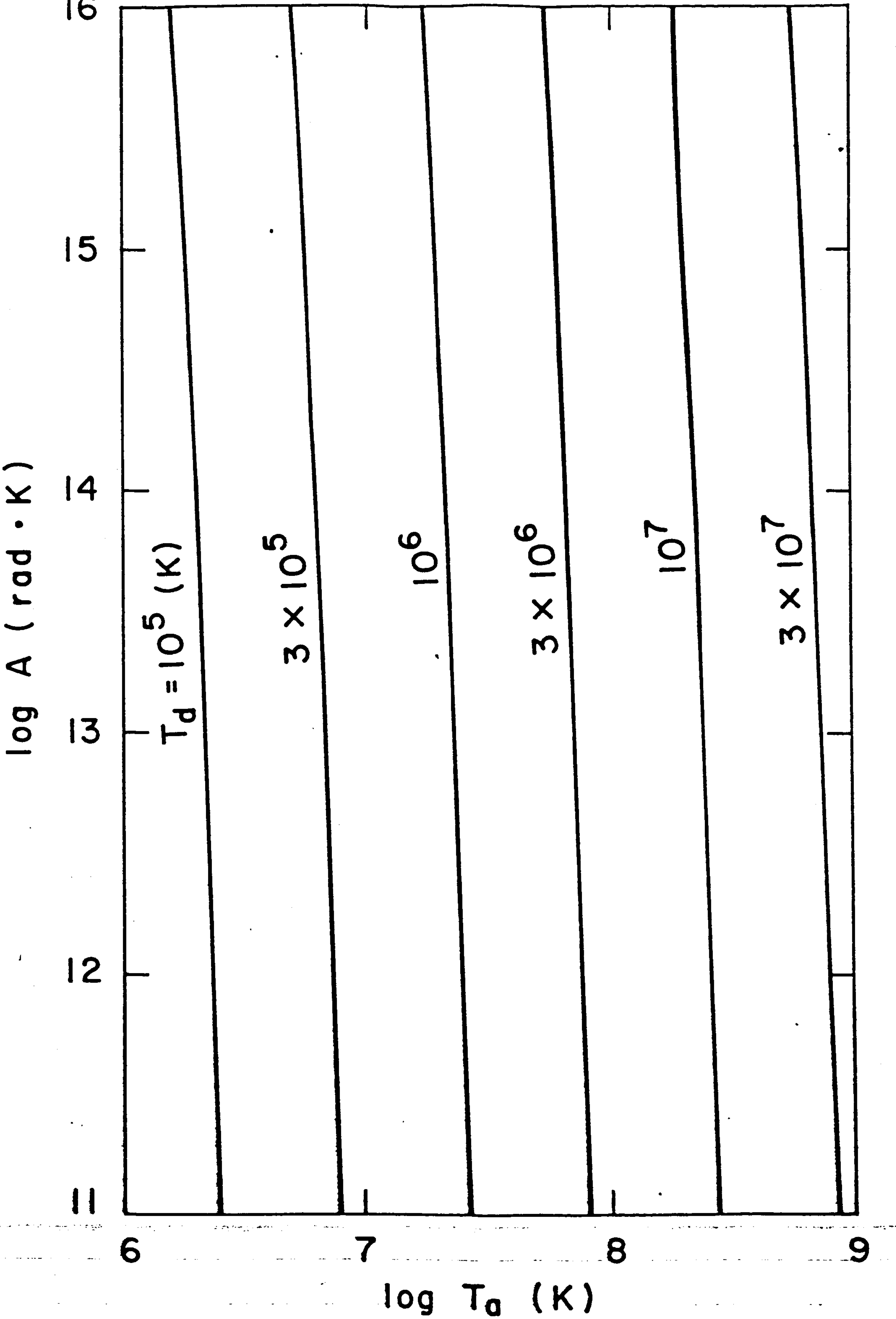
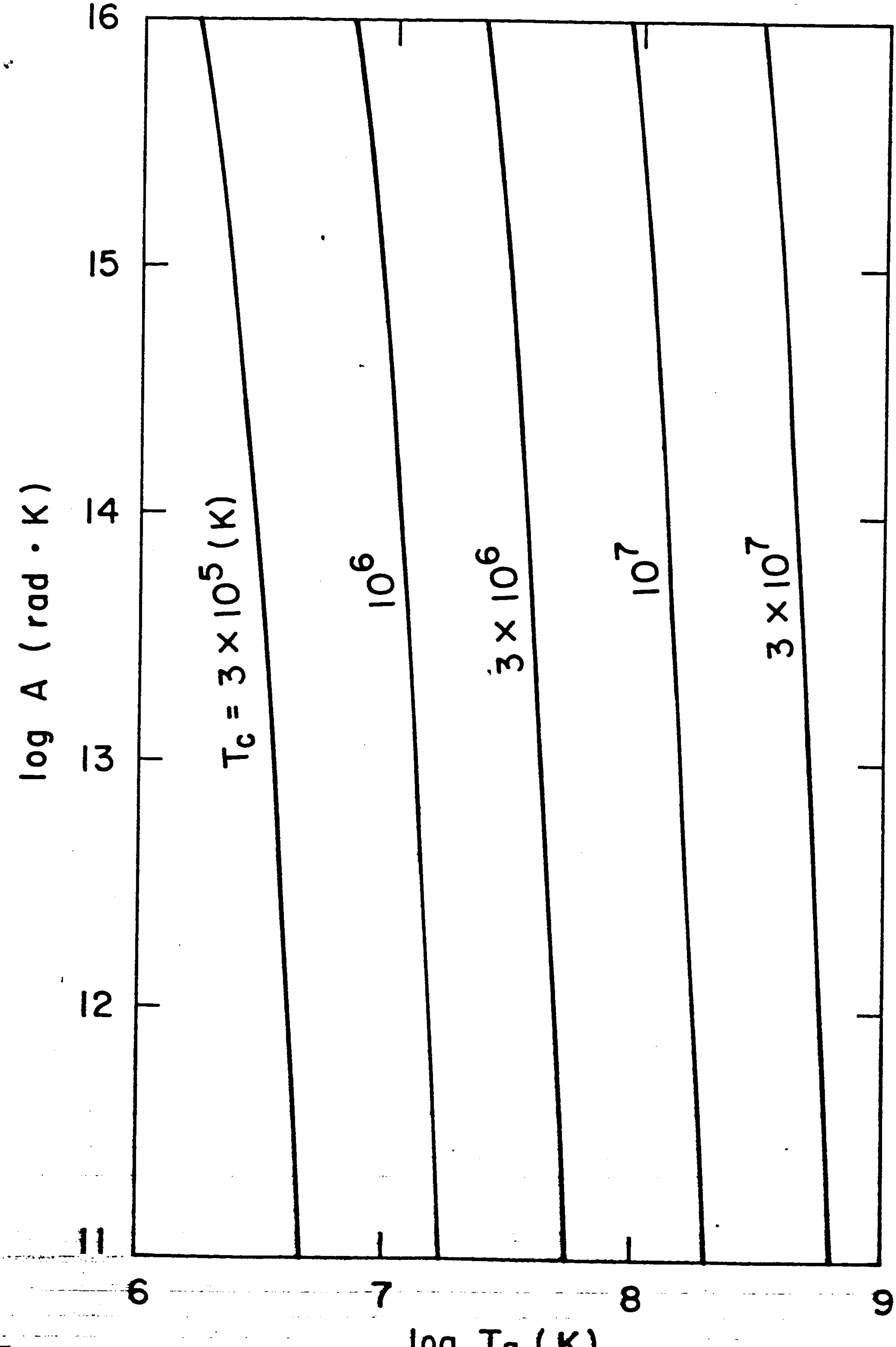

log A (rad · K)
16
15
14
13
12
11
6
7
8
9
log Ta (K)
Td = 10^5 (K)
3 × 10^5
10^6
3 × 10^6
10^7
3 × 10^7

Fig. 2



16
15
14
13
12
11
log A (rad · K)
$T_c = 3 \times 10^5$ (K)
10^6
3×10^6
10^7
3×10^7
6
7
8
9
log Ta (K)

Fig. 3